

A Mean-Field Description for AdS Black Hole

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In this paper we find an equivalent mean-field description for asymptotically AdS black hole in high temperature limit and in arbitrary dimensions. We obtain a class of mean-field potential for which the description is valid. We explicitly show that there is an one to one correspondence between the thermodynamics of a gas of interacting particles moving under a mean-field potential and an AdS black hole, namely the equation of state, temperature, pressure, entropy and enthalpy of both the systems match. In $3 + 1$ dimensions, in particular, the mean-field description can be thought of as an ensemble of tiny interacting *asymptotically flat* black holes moving in volume V and at temperature T . This motivates us to identify these asymptotically flat black holes as microstructure of asymptotically AdS black holes in $3 + 1$ dimensions.

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I. INTRODUCTION AND SUMMARY

Finding microscopic origin of black hole entropy is an unsolved problem in theoretical physics. A partial answer has been given in the context of string theory for a class of asymptotically flat supersymmetric black holes. However, a complete microscopic understanding for finite temperature asymptotically flat or AdS black hole is due.

An attempt to construct the microstructure of asymptotically AdS black hole in $3 + 1$ dimensions has been made in [1]. They introduced the idea of black hole molecules with some number densities to measure the microscopic degrees of freedom of the black hole and found that the number density suffers a sudden change accompanied by a latent heat when the black hole system undergoes a phase transition. They also found that there is a weak attractive interaction between two black hole molecules. From these two phenomena they tried to explore possible microscopic structure of a charged anti-de Sitter black hole completely from the thermodynamic

viewpoint. However, they could not find what the black hole microstates actually are. In this letter, we try to speculate about these black hole molecules.

A seemingly surprising similarity between phase diagrams of an AdS black hole and Van der Waals fluid was first observed in [2]. Considering the AdS black hole in a fixed electric charge ensemble (canonical) and identifying inverse temperature, electric charge and horizon radius of black hole with pressure, temperature and volume of liquid-gas system respectively, [2] showed that black hole phase diagram is similar to that of a non-ideal fluid described by the Van der Waals equation. However, ad-hoc identifications between the parameters on both sides were not clear. The story was modified by Kubiznak and Mann [3]¹, who, following [5], considered the negative cosmological constant to be thermodynamic pressure of the AdS black hole and volume covered by the event horizon to be thermodynamic volume conjugate to pressure². This allows one to set up an one to one correspondence between thermodynamic parameters on both sides. Although remarkable, the analogy between thermodynamics of charged AdS black holes and that of a Van der Waals fluid was only qualitative.

The Van der Waals equation, on the other hand, describes an equation of state of N interacting particles of mass m moving in a volume V at temperature T under mean field approximation and hence all the critical exponents of the system take the "mean field" value. Surprisingly, the critical exponents for AdS black hole evaluated from the equations of state also take the same "mean field" values. A natural question arises at this point if there is any effective mean field description for AdS black hole.

A related question has been attempted to answer in a recent beautiful paper by Rajagopal *et.al.*[7]. They constructed an asymptotically AdS black hole solution whose thermodynamics matches exactly that of a Van

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¹ See also [4]

² Phase diagram for dyonic black hole has been discussed in [6].

der Waals fluid³. However, the black hole solution they found is "unphysical" because, "the corresponding stress energy tensor does not obey any of the three standard energy conditions everywhere outside the horizon, though for certain values of the parameters it is possible to satisfy the energy conditions in a region near the horizon."

We pose a different question. *Can one construct an interacting system of particles under mean-field approximation such that, not only the equation of state, but the complete thermodynamics matches with that of an AdS black hole?* In this paper we attempt to answer this question. We find an interacting system of particles of mass $m^*(T)$ under mean field approximation in D space dimensions whose equation of motion, entropy, enthalpy and other thermodynamic quantities match with those of a $D + 1$ dimensional AdS black hole in *high temperature limit*. We also find a class of mean-field potentials for the system of particles. A particular example of such potential is plotted in figure 1.

In $D = 3$, in particular, it turns out that the mass of these interacting objects depends on temperature of the system as

$$m^*(T) \sim \frac{\hbar c^3}{kG_4 T} + \mathcal{O}\left(\frac{1}{T^2}\right). \quad (1)$$

This relation is exactly same as the relation between mass and temperature of an asymptotically flat neutral black hole in $3 + 1$ dimensions. Therefore, one can think of the particles are essentially tiny flat black holes moving in a volume V . Thus, we can identify the black hole molecules introduced in [1] as tiny asymptotically flat black holes. This motivates us to find an one to one correspondence between a gas of interacting flat black holes and AdS black hole in $3 + 1$ dimensions and hence the interacting gas system can be thought of as microstructure of AdS black hole. We summarise our result in following table.

Mean-field Theory (Interacting black holes in $3+1$ d flat space)	AdS Black Hole
Temperature	Temperature
Inverse density	Inverse charge density
Pressure	Cosmological constant
Entropy	Horizon area (entropy of black hole)
Enthalpy	Mass of black hole

In other dimensions the equivalence between gas of interacting particles and AdS black hole holds but the interpretation of interacting system in terms of an ensemble of flat black holes fails.

The plan of this paper is following. In sec. II we give a lightning review of classical cluster expansion in arbitrary dimensions. The mean-field description for $D + 1$ dimensional AdS black hole has been discussed in section III. Finally, in section IV we discuss some important features of our work.

II. NON-IDEAL GAS IN D DIMENSIONS AND CLASSICAL CLUSTER EXPANSION

In this section we briefly review the method of cluster expansion for classical non-ideal gas in D space dimension. We skip the details and quote only the main result. A complete discussion on cluster expansion can be found in [9].

Let us consider a classical system of N particles of effective mass $m^*(T)$ moving in a D dimensional volume V . The system is in contact with a heat bath of temperature T . The Hamiltonian is given by,

$$\mathcal{H} = \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m^*(T)} + \sum_{i<j} v_{ij} \quad (2)$$

where \mathbf{p}_i is total momentum of the i^{th} particle and $v_{ij} = v(|\mathbf{r}_i - \mathbf{r}_j|)$ is interaction potential between i^{th} particle and j^{th} particle. The effective mass $m^*(T)$ of these particles depends on temperature of the system. The partition function is given by,

$$\mathcal{Q}_N(V, T) = \frac{1}{N! h^{DN}} \int d^{DN} p \int_V d^{DN} r \exp \left[-\beta \sum_i \frac{p_i^2}{2m^*(T)} - \beta \sum_{i<j} v_{ij} \right].$$

The above integration can be simplified after doing the momentum integration,

$$\mathcal{Q}_N(V, T) = \frac{1}{N! \lambda^{DN}(T)} \int d^{DN} r \exp \left(-\beta \sum_{i<j} v_{ij} \right) \quad (3)$$

where $\lambda(T) = \sqrt{2\pi\hbar^2/m^*(T)kT}$ is thermal wavelength of these particles. The integral in eq.(3) is called the *configuration integral*. There is a well known systematic way of calculating this integral by expanding it in powers of $(e^{-\beta v_{ij}} - 1)$. This method is widely known as the **cluster expansion** of Ursell and Mayer[10]. The main application of this method is to calculate the higher order of virial coefficients for non-ideal gas.

Following [9], one can compute the grand canonical partition function in the limit $N \rightarrow \infty$

$$\log \mathcal{L}(z, V, T) = \frac{V}{\lambda^D} \sum_{l=1}^{\infty} b_l z^l \quad (4)$$

where, b_l 's are called *cluster integral* and defined as,

$$b_l(V, T) = \frac{1}{l! \lambda^{Dl-DV}} (\text{sum of all } l\text{-cluster}) \quad (5)$$

³ A generalisation to arbitrary dimensions has been constructed in [8].

and z is chemical potential. Note that the cluster integrals are dimensionless. In thermal equilibrium other thermodynamic quantities like pressure and density of the system can be written in terms of cluster integrals as,

$$\frac{P}{kT} = \frac{1}{\lambda^D} \sum_{l=1}^{\infty} b_l z^l, \quad \frac{N}{V} = \frac{1}{v} = \frac{1}{\lambda^D} \sum_{l=1}^{\infty} l b_l z^l, \quad (6)$$

where

$$b_l(T) \equiv \lim_{V \rightarrow \infty} b_l(V, T). \quad (7)$$

From eq.(6) one can find that the virial expansion of equation of state is given by,

$$\frac{Pv}{kT} = \sum_{l=1}^{\infty} a_l(T) \left(\frac{\lambda^D}{v} \right)^{l-1}, \quad (8)$$

where the virial coefficients a_l 's are determined in terms of cluster integrals from the following identity,

$$\begin{aligned} & (b_1 z + 2b_2 z^2 + 3b_3 z^3 + \dots) \left[a_1 + a_2 \left(\sum_{n=1}^{\infty} n b_n z^n \right) \right. \\ & \left. + a_3 \left(\sum_{n=1}^{\infty} n b_n z^n \right)^2 + \dots \right] = b_1 z + b_2 z^2 + b_3 z^3 + \dots \end{aligned}$$

First few of them are given by,

$$\begin{aligned} a_1 &= b_1 = 1, \quad a_2 = -b_2, \quad a_3 = 4b_2^2 - 2b_3, \\ a_4 &= -20b_2^3 + 18b_2 b_3 - 3b_4, \quad \dots \end{aligned} \quad (9)$$

The entropy of this system is given by,

$$\begin{aligned} S &= kT \left(\frac{\partial \log \mathcal{L}}{\partial T} \right)_{z,V} - Nk \log(z) + k \log \mathcal{L} \\ &= \frac{kV}{\lambda^D} \left[\left(1 - \frac{DT}{\lambda} \frac{d\lambda}{dT} \right) \mathcal{B} + T \frac{d\mathcal{B}}{dT} \right] - Nk \log(z) \end{aligned} \quad (10)$$

where,

$$\mathcal{B} = \sum_{l=1}^{\infty} b_l z^l. \quad (11)$$

The enthalpy is given by,

$$\begin{aligned} H &= U + kT \log \mathcal{L} \\ &= kT^2 \left(\frac{\partial \log \mathcal{L}}{\partial T} \right)_{z,V} + kT \log \mathcal{L} \\ &= \frac{kTV}{\lambda^D} \left[\left(1 - \frac{DT}{\lambda} \frac{d\lambda}{dT} \right) \mathcal{B} + T \frac{d\mathcal{B}}{dT} \right]. \end{aligned} \quad (12)$$

III. $D+1$ DIMENSIONAL ELECTRICALLY CHARGED ADS BLACK HOLE

In this section we derive the equation of state of a $D+1$ dimensional electrically charged AdS black hole and find

an equivalent system of interacting particles under mean-field approximation whose equation of state, entropy and enthalpy match with those of black hole.

We consider Reissner-Nordstrom action in $D+1$ dimensions in presence of a cosmological constant $\Lambda = -\frac{D(D-1)}{2b^2}$

$$I = \frac{c^3}{16\pi G_{D+1}} \int d^{D+1}x \sqrt{-g} \left[R - \frac{G_{D+1} \epsilon_0^{(D)}}{c^2} F^2 + \frac{D(D-1)}{b^2} \right] \quad (13)$$

where, G_{D+1} and $\epsilon_0^{(D)}$ are Newton's constant and permittivity in $D+1$ dimension. The equations of motion are given by,

$$\begin{aligned} R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R - \frac{D(D-1)}{2b^2} g_{\mu\nu} &= \frac{2G_{D+1} \epsilon_0^{(D)}}{c^2} (F_{\mu\lambda} F_{\nu}^{\lambda} - \frac{g_{\mu\nu}}{4} F^2), \\ \nabla_{\mu} F^{\mu\nu} &= 0. \end{aligned} \quad (14)$$

A static, spherically symmetric, electrically charged black hole solution is given by,

$$A = \frac{1}{\epsilon_0^{(D)} c \eta} \left(-\frac{q_e}{r^{D-2}} + \frac{q_e}{r_+^{D-2}} \right) c dt, \quad (15)$$

$$ds^2 = -f(r) c^2 dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (16)$$

where,

$$f(r) = \left(1 + \frac{r^2}{b^2} - \frac{G_{D+1}}{c^2} \frac{2m}{r^{D-2}} + \frac{G_{D+1}}{\epsilon_0^{(D)} c^4} \frac{q_e^2}{r^{2D-4}} \right). \quad (17)$$

q_e and m are integration constants identified as electric charge and mass parameters of the black hole. r_+ is horizon radius. Physical mass and charge are given by,

$$M = \frac{(D-1)S_{D-1}}{8\pi} m, \quad Q = \sqrt{2(D-1)(D-2)} \frac{S_{D-1}}{8\pi} q_e. \quad (18)$$

The asymptotic value of gauge field is identified with the chemical potential corresponding to charge q_e

$$\phi_e = \frac{1}{\epsilon_0^{(D)} \eta} \frac{q_e}{r_+^{D-2}}. \quad (19)$$

Following [3] we consider the cosmological constant as thermodynamical pressure of the system

$$P = -\frac{c^4}{G_{D+1}} \frac{\Lambda}{8\pi} = \frac{D(D-1)c^4}{16\pi G_{D+1}} \frac{1}{b^2}. \quad (20)$$

The hawking temperature of this black hole is given by

$$kT = \frac{(D-2)\hbar c}{4\pi r_+} \left(1 + \frac{16\pi G_{D+1}}{(D-1)(D-2)c^4} P r_+^2 - \frac{G_{D+1} \epsilon_0^{(D)}}{c^4} \phi_e^2 \right). \quad (21)$$

We consider our system in contact with a reservoir at fixed temperature and chemical potential. Thermodynamic variables associated with our system are energy E , pressure P and charge. Then the first law of black hole thermodynamics and Helmholtz potential W are given by,

$$dE = TdS - PdV + \phi_e dq_e, \quad W = E - TS - \phi_e q_e. \quad (22)$$

We refer to [2, 6] for detailed computation of thermodynamic potential and variables. Here we quote the final results. The free energy is given by,

$$W = \frac{I}{\beta_t} = \frac{S_{D-1}c^4}{16\pi G_{D+1}} \left[-\frac{G_{D+1}\epsilon_0^{(D)}}{c^4} \phi_e^2 r_+^D - \frac{16\pi G_{D+1}P}{D(D-1)c^4} r_+^D + r_+^{D-2} \right] \quad (23)$$

where, β_t is periodicity of Euclidean time direction. $\beta_t = \hbar\beta = \hbar/kT$. Corresponding thermodynamic variables can be computed from the free energy,

$$\langle q \rangle = -\frac{\partial W}{\partial \phi_e} = \frac{(D-2)S_{D-1}}{4\pi} \epsilon_0^{(D)} \phi_e r_+^{D-2}, \quad (24)$$

$$V_{bh} = \frac{\partial W}{\partial P} = \frac{S_{D-1}}{D} r_+^D, \quad (25)$$

$$S_{bh} = -\frac{\partial W}{\partial T} = \frac{c^3 k}{4\hbar G_{D+1}} S_{D-1} r_+^{D-1} = \frac{S_{D-1} k r_+^{D-1}}{4l_p^{D-1}} \quad (26)$$

where l_p is the Planck's length in D dimensions. From Helmholtz free energy we can compute the enthalpy or total energy of the system as,

$$H_{bh} = W + TS + \phi_e q_e = Mc^2. \quad (27)$$

Defining $V_o = DV_{bh}/S_{D-1}$, in the limit of large black hole temperature black hole enthalpy and enthalpy can be written as,

$$H_{bh} = \frac{S_{D-1}}{4l_p^{D-1}} kT_r V_o^{\frac{D-1}{D}}, \quad S_{bh} = \frac{S_{D-1}}{4l_p^{D-1}} kV_o^{\frac{D-1}{D}} \quad (28)$$

where

$$T_r = \frac{(D-1)}{D} T \quad (29)$$

is the reduced temperature.

A. Equation of state

In this subsection we derive the equation of state of an electrically charged AdS black hole in $D+1$ dimensions. In thermal equilibrium the equation of state gives a relation between pressure, temperature and volume (or density). At high temperature and large volume limit (classical limit) the equation of state can be written in powers of inverse temperature and inverse volume.

We first find black hole pressure in terms of volume and temperature by using eq.(21) and eq.(25),

$$P = \frac{d}{4l_p^{D-1}} \frac{kT_r}{V_o^{1/D}} - \frac{(D-1)(D-2)}{16\pi} \frac{kT_p}{l_p^{D-2}} \left(1 - \frac{\phi^2}{\phi_p^2} \right) \frac{1}{V_o^{2/D}} \quad (30)$$

where, $T_p = \hbar c/k l_p$ is Planck's temperature, $\phi_p = \frac{q_p}{\epsilon_0^{(D)} l_p^{D-2}}$ and $q_p = \sqrt{\epsilon_0^{(D)} \hbar c l_p^{D-3}}$ is Planck's charge in D dimensions.

Before we proceed to write the equation of state we notice, from eq. (28), that both the enthalpy and entropy are proportional to $V_o^{(D-1)/D}$, in the large volume limit. This is because of the holographic nature of black hole. Unlike usual thermodynamic objects entropy and other extensive thermodynamic quantities of black hole do not depend on volume rather they depend on the volume of a lower dimensional hypersurface (horizon). Motivated by this, we define a *reduced volume* of the system,

$$V_r = l_p V_o^{(D-1)/D}. \quad (31)$$

We multiply $V_o^{(D-1)/D}$ by a factor of l_p such that the reduced volume has the correct dimensions. The entropy and enthalpy written in term of reduced parameters are given by

$$S_{bh} = \frac{S_{D-1} k V_r}{4l_p^D}, \quad H_{bh} = \frac{S_{D-1} k T_r V_r}{4l_p^D}. \quad (32)$$

We further define a quantity reduced volume per unit charge number as

$$v_r = \frac{V_r}{\bar{q}} = \left(\frac{\phi_p l_p^{D-1}}{\phi_e} \right) r_+, \quad \bar{q} = \frac{q_e}{q_p} = \text{charge number}. \quad (33)$$

In the thermodynamic limit, v_r is also very large hence $1/v_r$ can be considered as an expansion parameter. Redefining the pressure,

$$P_r = \frac{4\phi_e}{D\phi_p} P, \quad (34)$$

we can write the equation of states in terms of reduced variables as

$$\frac{P_r v_r}{kT_r} = 1 - \left[\frac{(D-1)(D-2)T_p}{4\pi D T_r} \left(\frac{\phi_p}{\phi_e} - \frac{\phi_e}{\phi_p} \right) l_p^D \right] \left(\frac{1}{v_r} \right). \quad (35)$$

To match this equation with virial expansion of pressure (eq.8), we introduce black hole thermal wavelength $\lambda_b(T_r)$ and write the equation of state as,

$$\frac{P_r v_r}{kT_r} = 1 - \left[\frac{(D-1)(D-2)T_p}{4\pi D T_r} \left(\frac{\phi_p}{\phi_e} - \frac{\phi_e}{\phi_p} \right) \frac{l_p^D}{\lambda_b^D} \right] \left(\frac{\lambda_b^D}{v_r} \right). \quad (36)$$

Unlike eq.(8) this is not an infinite series, the expansion stops at $l = 2$.

In [3] the horizon radius, r_+ , was identified with the volume per particle, v , of a Van der Walls fluid. This identification was some what ad-hoc. Here we see that v_r which is reduced volume per unit *charge number* is proportional to r_+ and we identify this inverse charge number density with the inverse density of Van der Walls fluid.

Our next goal is to set up a dictionary between a thermodynamic system of *AdS* black hole and a system of N interacting gas of particles in volume V and temperature T such that the equation of state, entropy and enthalpy of these two systems match.

B. The dictionary

Comparing the equation of state of a charged *AdS* black hole and that of a non-ideal gas, we find the following identifications

$$P_r = P, \quad v_r = v, \quad T_r = T \quad \text{and} \quad \lambda_b(T_r) = \lambda(T). \quad (37)$$

The virial coefficients are given by,

$$a_2(T) = -\frac{(D-1)(D-2)T_p}{4\pi DT_r} \left(\frac{\phi_p}{\phi_e} - \frac{\phi_e}{\phi_p} \right) \frac{l_p^D}{\lambda_b^D},$$

$$a_l(T) = 0, \quad l \geq 3. \quad (38)$$

Therefore, the cluster integrals are given by

$$\bar{b}_2(T) = \frac{(D-1)(D-2)T_p}{4\pi DT_r} \left(\frac{\phi_p}{\phi_e} - \frac{\phi_e}{\phi_p} \right) \frac{l_p^D}{\lambda_b^D}, \quad (39)$$

$$\bar{b}_3(T) = 2\bar{b}_2^2(T), \quad \bar{b}_4(T) = \frac{16}{3}\bar{b}_2^3(T) \quad \dots \quad (40)$$

Equating enthalpy of black hole with that of a non-ideal gas we get,

$$\left(1 - \frac{DT}{\lambda_b(T)} \frac{d\lambda_b(T)}{dT} \right) \mathcal{B} + T \frac{d\mathcal{B}}{dT} = \frac{S_{D-1}\lambda_b^D(T)}{4l_p^D}. \quad (41)$$

Finally, equating entropy on both sides we find,

$$z = 1. \quad (42)$$

Thus the system of N interacting particles, which is equivalent to an *AdS* black hole with constant chemical potential, has constant fugacity.

Eq.(41) along with the values of cluster integrals given in eq.(39) can be solved to find $\bar{b}_2(T)$ and hence the temperature dependence of the effective mass of an equivalent system of non-ideal gas of particles. Eq. (41) is highly non-linear and difficult to solve exactly. However, in the limit of high temperature this equation can be solved and one finds the temperature dependence of black hole thermal wavelength (or effective mass) order by order in $1/T$ expansion. At high temperature \bar{b}_2 is small and hence we can neglect the terms proportional to higher powers of \bar{b}_2 . Thus, solving the above differential equation the thermal wavelength turns out to be

$$\lambda_b(T) \sim l_p + \mathcal{O}\left(\frac{1}{T}\right), \quad (43)$$

which implies the effective mass of the particles goes as

$$m^*(T) = \frac{\hbar^2}{kT} \left(\frac{c^3}{\hbar G_{D+1}} \right)^{\frac{2}{D-1}}. \quad (44)$$

In four spacetime dimensions ($D = 3$) the relation becomes,

$$m^*(T) \sim \frac{\hbar c^3}{kG_4 T} + \mathcal{O}\left(\frac{1}{T^2}\right). \quad (45)$$

This relation is exactly same as the relation between mass and temperature of an asymptotically flat neutral black hole in four dimensions. Therefore, we can think of equations of state, entropy, enthalpy of an interacting gas of asymptotically flat 4 dimensional black holes in volume V and at temperature T are same as those of an *AdS* black hole in constant electric potential ensemble. Thus, we can identify these asymptotically flat black holes as black hole molecules introduced in [1]. See section IV for a detailed discussion.

C. The mean-field potential

In this subsection construct the mean field potential in which the gas of flat black holes are moving. One should note that from Eqn. (39) that,

$$\bar{b}_2(T)\lambda_b^D = \frac{\alpha}{T} \quad (46)$$

where,

$$\alpha = \frac{(D-1)(D-2)T_p}{4\pi} \left(\frac{\phi_p}{\phi_e} - \frac{\phi_e}{\phi_p} \right) l_p^D.$$

\bar{b}_2 , which is proportional to sum over all 2 clusters (eq.(5)), is given by

$$\bar{b}_2(T)\lambda(T)^D = \frac{S_{D-1}}{2} \int_0^\infty r^{D-1} dr \left(e^{-\beta u(r)} - 1 \right) = \frac{\alpha}{T}. \quad (47)$$

Potentials satisfying the above condition will give thermodynamic/microscopic description of the *AdS* black hole at high temperature. One can choose the following ansatz for the potential,

$$u(r) = \begin{cases} u_0 & r < r_0 \\ -u_1 \left(\frac{r_0}{r} \right)^n & r \geq r_0. \end{cases} \quad (48)$$

To get finite result, $n \geq D$. Assuming $u_0\beta \ll 1$ and $u_1\beta \ll 1$ we find

$$\frac{1}{D} \left(u_0 - \frac{Du_1}{n-D} \right) r_0^D = -\frac{2\alpha k}{S_{D-1}}. \quad (49)$$

The minimum distance of approach r_0 can be taken to be twice the size of a flat black hole.

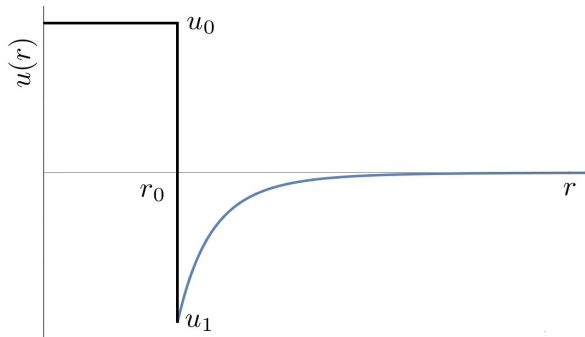


FIG. 1: A mean-field potential for AdS black hole

IV. DISCUSSION

We find an equivalent system of N interacting particles under mean-field approximation whose thermodynamics and equation of state match exactly with those of an electrically charged AdS black hole in high temperature limit. This qualitative thermodynamic equivalence between these two systems holds in any arbitrary dimensions. However, in four spacetime dimensions, in particular, the interacting system of particles can be thought of as a classical ensemble of large number of tiny, interacting asymptotically flat black holes moving in volume V . We consider these black holes as objects/particles moving under a mean-field potential $u(r)$. Although, these micro black holes themselves have micro structure, but we do not talk about those structure, rather we consider them

almost point like objects carrying the degrees of freedom of an AdS black hole. The classical cluster expansion is valid if $\lambda_b^3/v_r \ll 1$, that is average inter black hole distance is much much bigger than the Planck length.

A possible microscopic structure of a $3+1$ dimensional, charged, anti-de Sitter black hole has been proposed in [1] from the thermodynamic viewpoint. They introduced black hole molecules as microstructure with a number density and studied how the number density changes under phase transition between small black hole and big black hole. They also considered the interaction between these molecules. However, the exact microstructure was unknown to them. In this paper we find an one to one correspondence between a gas of asymptotically flat black holes and AdS black hole and hence we identify these flat black holes as black hole molecules (as introduced in [1]) of charged AdS black holes in $3+1$ dimensions. The logarithm of phase space volume covered by these tiny black holes proportional to entropy (horizon area) of AdS black hole. Note that, our argument is valid at high temperature only where one can treat the system classically.

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